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Lesson 6: Exponential Growth—U.S. Population and World Population

Student Outcomes

* Students compare linear and exponential models of population growth.

Classwork

Example 1 (8 minutes)

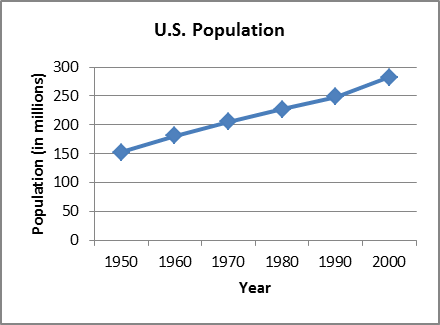
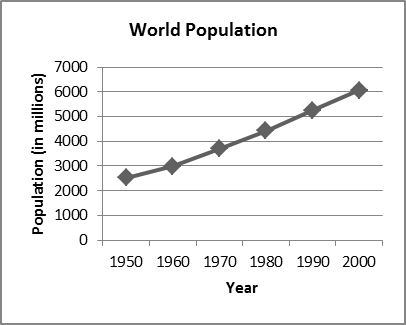
Give students time to review the two graphs. Ask students to compare the two graphs and make conjectures about the rates of change. As the students share the conjectures with the class, respond without judgment as to accuracy, simply suggesting that we will investigate further and see which conjectures are correct.

Example 1

Callie and Joe are examining the population data in the graphs below for a history report. Their comments are as follows:

Callie: It looks like the U.S. population grew the same amount as the world population, but that can’t be right, can it?

Joe: Well, I don’t think they grew by the same *amount*, but they sure grew at about the same rate. Look at the slopes.



Be aware that students frequently ignore scale on graphs and may offer incorrect observations as a result. If students respond incorrectly to the prompts, direct partners or groups to discuss why the response or observation is incorrect.

* 1. Is Callie’s observation correct? Why or why not?

No. The world population grew by a far greater amount as shown by the scale of the vertical axis.

* 1. Is Joe’s observation correct? Why or why not?

No. Again, Joe ignored the scale, just as Callie did. The rate of change (or slope) is much greater for the world population than for the U.S. population.

* 1. Use the World Population graph to estimate the percent increase in world population from 1950 to 2000.

Using million for the year 1950 and million for the year 2000 gives a percent increase of , obtained by computing .

* 1. Now use the U.S. Population graph to compute the percent increase in the U.S. population for the same time period.

Using million for the year 1950 and million for the year 2000 gives a percent increase of , obtained by computing .

* 1. How does the percent increase for the world population compare to that for the U.S. population over the same time period, 1950 to 2000?

The world population was increasing at a faster average rate than the U.S. population was.

* 1. Do the graphs above seem to indicate linear or exponential population growth? Explain your response.

In the time frame shown, the growth appears to be linear. The world population is increasing at an average rate of about million per year. The U.S. population is increasing at an average rate of about million per year.

* 1. Write an explicit formula for the sequence that models the world population growth from 1950–2000 based on the information in the graph. Assume that the population (in millions) in 1950 was and in 2000 was . Use to represent the number of years after 1950.

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Example 2 (15 minutes)

Ask students to compare the graph below with the World Population graph Callie and Joe were using in Example 1. Again, students may respond incorrectly if they ignore scale. Requiring students to investigate and discover why their responses are incorrect will result in deeper understanding of the concept.

Joe tells Callie he has found a different world population graph that looks very different from their first one.

**Example 2**

* 1. How is this graph similar to the World Population graph in Example ? How is it different?

This graph uses the same vertical scale as the one for world population in Example 1. This graph is different in two ways: 1. It shows years from AD 1700 through 2000 instead of from1950-2000, and 2. The graph itself shows that population growth took an exponential turn in approximately 1850.

* 1. Does the behavior of the graph from 1950–2000 match that shown on the graph in Example?

Yes, both graphs show that 1950 world population was about million and 2000 world population was just over million.

* 1. Why is the graph from Example 1 somewhat misleading?

The graph in Example makes it appear as if the world population has grown in a linear fashion when it has really grown exponentially if examined over a longer period of time.

* 1. **An exponential formula that can be used to model the world population growth from 1950 through 2000 is as follows:**  
        
       
     where represents the world population in the year 1950, and represents the number of years after 1950. Use this equation to calculate the world population in 1950, 1980, and 2000. How do your calculations compare with the world populations shown on the graph?

. The amounts are similar to those shown on the graph.

* 1. Following is a table showing the world population numbers used to create the graphs above.

|  |  |
| --- | --- |
| Year | World Population  (in millions) |
| 1700 |  |
| 1750 |  |
| 1800 |  |
| 1850 |  |
| 1900 |  |
| 1950 |  |
| 1960 |  |
| 1970 |  |
| 1980 |  |
| 1990 |  |
| 2000 |  |

How do the numbers in the table compare with those you calculated in part (d) above?

1950 is identical (since it was used as the base year); 1980 is reasonably close (vs. million; about variance); 2000 is very close (within million; about % variance).

* 1. **How is the formula in part (d) above different from the formula in Example 1 part (g)? What causes the difference? Which formula more closely represents the population?**

The formula in Example 1 part (g) is linear while the formula in part (d) above is exponential. The growth rate in the linear formula is a fixed million increase in population each year whereas the growth rate in the exponential formula is a factor of or An exponential equation grows by a constant factor each year, while a linear equation grows by a constant difference each year. The exponential equation offers a more accurate model since the projected population numbers using this model more closely match the actual figures.

Exercises (17 minutes)

Have students work with a partner or small group to answer the exercises. Circulate to respond to group questions and to guide student responses.

Exercises

|  |  |
| --- | --- |
| Year | U.S. Population  (in millions) |
| 1800 |  |
| 1900 |  |
| 2000 |  |

1. The table below represents the population of the U.S. (in millions) for the specified years.   
     
     
     
     
     
     
     
     
     
     
     
     
     
     
   1. If we use the data from 1800–2000 to create an exponential equation representing the population, we generate the following formula for the sequence, where represents the U.S. population and represents the number of years after 1800.  
         
        
      Use this formula to determine the population of the U.S. in the year 2010.

This formula yields a U.S. population of million in 2010.

* 1. If we use the data from 1900–2000 to create an exponential formula that models the population, we generate the following, where represents the U.S. population and represents the number of years after 1900.  
        
       
     Use this formula to determine the population of the U.S. in the year 2010.

***This formula yields a U.S. population of million in 2010.***

* 1. The actual U.S. population in the year was million. Which of the above formulas better models the U.S. population for the entire span of 1800–2010? Why?

***The formula in part (b) resulted in a closer approximation of the 2010 population. Although the population of the U.S. is still increasing exponentially, the rate has slowed considerably in the last few decades. Using the population from 1800–2000 to generate the formula results in a growth factor higher than the rate of the current population growth.***

* 1. Complete the table below to show projected population figures for the years indicated. Use the formula from part (b) to determine the numbers.

|  |  |
| --- | --- |
| Year | World Population  (in millions) |
| 2020 |  |
| 2050 |  |
| 2080 |  |

* 1. Are the population figures you computed reasonable? What other factors need to be considered when projecting population?

***These numbers do not necessarily take into account changes in technology, efforts to reduce birth rates, food supply, or changes in life expectancy due to disease or scientific advances. Students may come up with a variety of responses.***

1. The population of the country of Oz was in the year 2010. The population is expected to grow by a factor of annually. The annual food supply of Oz is currently sufficient for a population of people and is increasing at a rate which will supply food for an additional people per year.
   1. Write a formula to model the population of Oz. Is your formula linear or exponential?

**, with representing population and representing years after 2010. The formula is exponential.**

* 1. Write a formula to model the food supply. Is the formula linear or exponential?

**, with representing the food supply in terms of number of people supplied with food and representing the number of years after 2010. The equation is linear.**

* 1. At what point does the population exceed the food supply? Justify your response.

**The population exceeds the food supply sometime during 2015. Students might use a table or a graph to support this response.**

* 1. **If Oz doubled its current food supply (to million), would shortages still take place? Explain.**

**Yes; the food supply would run out during the year 2031. Again, students may justify their response using with a graph or a table.**

* 1. **If Oz doubles both its beginning food supply and doubles the rate at which the food supply increases, would food shortages still take place? Explain.**

**Yes; the food supply would run out in the year 2034. Students may justify with either a graph or a table.**

Closing (2 minutes)

* Why did the equation increase so much more quickly that the equation ?
  + The first formula is exponential while the second formula is linear.
* One use of studying population growth involves estimating food shortages. Why might we be interested in modeling population growth at a local level?
  + *City planners may use population models to plan for road construction, school district boundaries, sewage and water facilities, and similar infrastructure issues.*

Exit Ticket (3 minutes)

Name \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

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Exit Ticket

Do the examples below require a linear or exponential growth model? State whether each example is linear or exponential, and write an explicit formula for the sequence that models the growth for each case. Include a description of the variables you use.

1. A savings account accumulates no interest but receives a deposit of per month.

1. The value of a house increases by per year.
2. Every year, the alligator population is of the previous year’s population.
3. The temperature increases by every minutes from a.m. to p.m. each day for the month of July.
4. Every 240 minutes, of the rodent population dies.

Exit Ticket Sample Solutions

Do the examples below require a linear or exponential growth model? State linear or exponential for each, and write an explicit formula that models the growth for each case. Include a description of the variables you use.

1. A savings account, accumulating no interest with a deposit of per month.

**linear; , where represents the accumulated value in the account after months**

1. The value of a house increases by per year.

**exponential; , where represents the beginning value of the house and is the value of the house after years.**

1. Every year, the alligator population is of the previous year’s population.   
   **exponential; , where represents the current population of alligators and is the alligator population after years.**
2. The temperature increases by every minutes from a.m. to p.m. each day for the month of July.

***linear; ; where represents the beginning temperature and is the temperature after half-hour periods since***

1. Every 240 minutes, of the rodent population dies.  
   **exponential; ; where is the current population of rodents is the remaining population of rodents after four-hour periods.**

Problem Set Sample Solutions

1. Student Friendly Bank pays a simple interest rate of 2.5% per year. Neighborhood Bank pays a compound interest rate of 2.1% per year, compounded monthly.  
   1. Which bank will provide the largest balance if you plan to invest for 10 years? For 20 years?

***Student Friendly Bank gives a larger balance at the -year mark. Neighborhood Bank gives a larger balance by the -year mark.***

* 1. Write an explicit formula for the sequence that models the balance of the Student Friendly Bank balance, years after a deposit is left in the account.
  2. Write an explicit formula for the sequence that models the balance at the Neighborhood Bank balance, months after a deposit is left in the account.
  3. Create a table of values indicating the balances in the two bank accounts from year 2 to year 20 in 2 year increments. Round each value to the nearest dollar.

|  |  |  |
| --- | --- | --- |
| Year | Student Friendly Bank  (in dollars) | Neighborhood Bank  (in dollars) |
| 0 |  |  |
| 2 |  |  |
| 4 |  |  |
| 6 |  |  |
| 8 |  |  |
| 10 |  |  |
| 12 |  |  |
| 14 |  |  |
| 16 |  |  |
| 18 |  |  |
| 20 |  |  |

* 1. Which bank is a better short-term investment? Which bank is better for those leaving money in for a longer period of time? When are the investments about the same?

***Student Friendly Bank; Neighborhood Bank; they are about the same by the end of year 17.***

* 1. What type of model is Student Friendly Bank? What is the rate or ratio of change?

***linear; per year***

* 1. What type of model is Neighborhood Bank? What is the rate or ratio of change?

***exponential; per month***

1. The table below represents the population of the state of New York for the years 1800–2000. Use this information to answer the questions.

|  |  |
| --- | --- |
| Year | Population |
| 1800 |  |
| 1900 |  |
| 2000 |  |

* 1. Using the year 1800 as the base year, an explicit formula for the sequence that models the population of New York is **, where is the number of years after 1800.**   
     Using this formula, calculate the projected population of New York in 2010.
  2. Using the year 1900 as the base year, an explicit formula for the sequence that models the population of New York is **, where is the number of years after 1900.**Using this equation, calculate the projected population of New York in 2010.
  3. Using the internet (or some other source), find the population of the state of New York according to the 2010 census. Which formula yielded a more accurate prediction of the 2010 population?

***The actual population of the state of New York in 2010 was 19,200,000. The formula in part (b) resulted in a more accurate prediction.***